CMSC201
Computer Science I for Majors

Lecture 23 – Algorithmic Analysis

Last Class We Covered

- How data is represented and stored in memory
  - Binary numbers
  - Hexadecimal numbers
  - Converting
    - Binary to Decimal
    - Decimal to Binary
  - ASCII

Any Questions from Last Time?

Exercise: Converting

- What are the decimal/binary equivalents of…

0101

1001

0100 0110

151

227

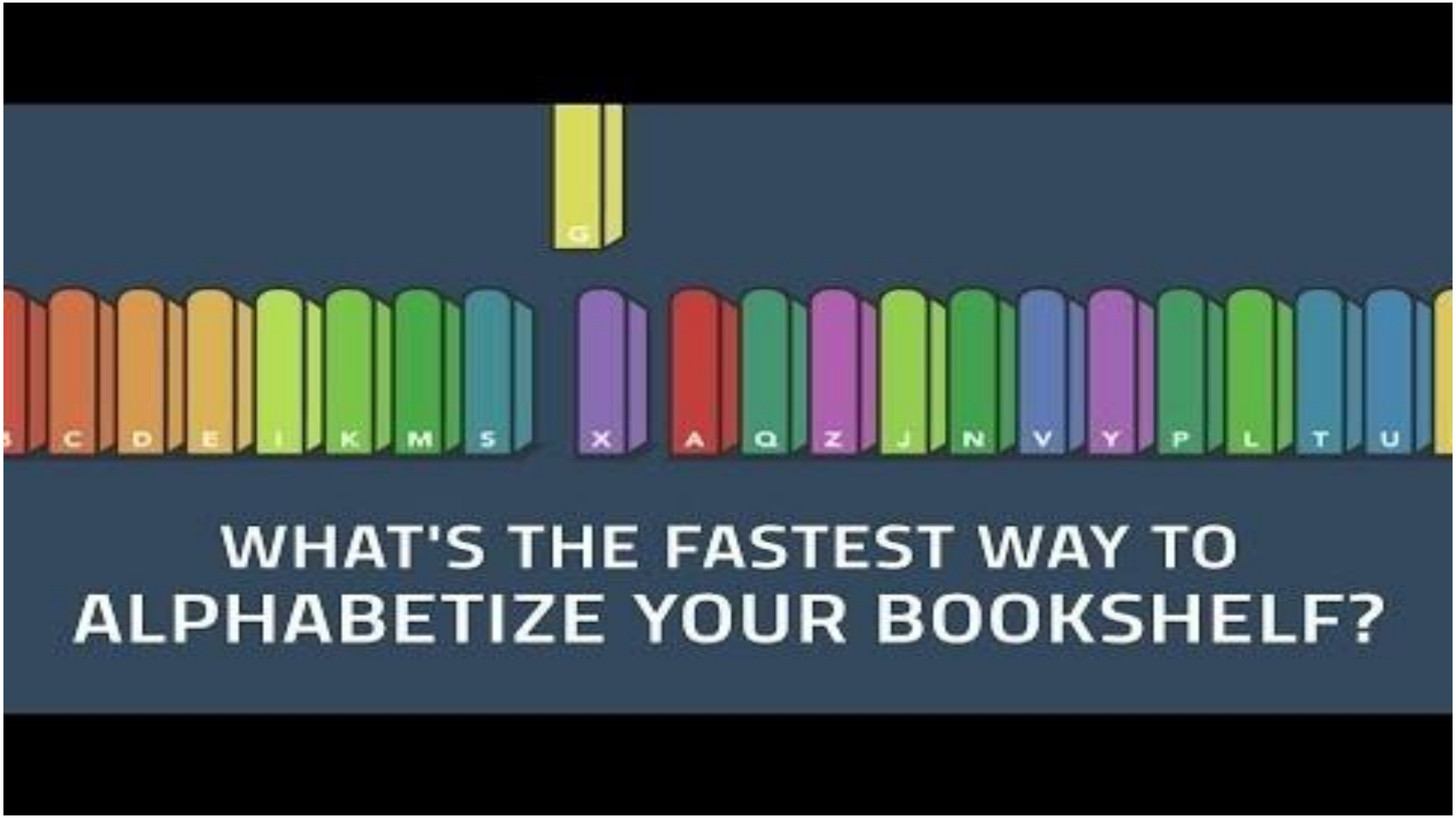
68

And what is this last one from hex to binary?

Today's Objectives

- To learn about asymptotic analysis
  - What it is
  - Why it's important
  - How to calculate it
- To discuss "run time" of algorithms
  - Why one algorithm is "better" than another

Alphabetizing a Bookshelf



Video from https://www.youtube.com/watch?v=WaNLJf8xzC4

Run Time

Run Time

- An algorithm's ***run time*** is the amount of "time" it takes for that algorithm to run
  - "Time" normally means number of operations or something similar, and not seconds or minutes

- Run time is shown as an expression, which updates based on how large the problem is
- Run time shows how an algorithm *scales*, or changes with the size of the problem

Example: Fibonacci Recursion

- Ideally, we want an algorithm that runs in a reasonable amount of time, no matter how large the problem

- Remember the recursive Fibonacci program?
  - It runs within one second for smaller positions
  - But the larger the position we ask for, the longer and longer it takes

Fibonacci Recursion

```
python fibEx.py (with position < 30):
   < 1 second

python fibEx.py (with position = 30):
   2 seconds

python fibEx.py (with position = 35):
   8 seconds

python fibEx.py (with position = 40):
   76 seconds
```


Fibonacci Recursion

`python fibEx.py (with position = 50):`

`Guess!`

`9,493 seconds`

`2 hours, 38 minutes, 13 seconds!!!`

Run Time for Linear Search

- Say we have a list that does not contain what we're looking for.
- How many things in the list does linear search have to look at for it to figure out the item's not there for a list of 8 things?
- 16 things?
- 32 things?

Run Time for Binary Search

- Say we have a list that <u>does not</u> contain what we're looking for.
- What about for binary search?
  - How many things does it have to look at to figure out the item's not there for a list of 8 things?
  - 16 things?
  - 32 things?
- Notice anything different?

Different Run Times

- These algorithms scale differently!
  - Linear search does an amount of work equal to the number of items in the list
  - Binary search does an amount of work equal to the $\mathtt{log}_2$ of the numbers in the list!
- By the way, $\mathtt{log}_2\mathtt{(x)}$ is basically asking "2 to what power equals x?" (normally shown as **`lg(x)`**)
  - This is the same as saying, "how many times must we divide x in half before we hit 1?"

Bubble Sort Run Time

- For a list of size **N**, how much work do we do for a single pass?
  - **N**
- How many passes will we have to do?
  - **N**
- What is the run time of Bubble Sort?
  - $\mathbf{N}^2$

Selection Sort Run Time

- What is the run time of finding the lowest number in a list?

- For a list of size **N**, how many elements do you have to look through to find the min?
- **N**

Selection Sort Run Time

- For a list of size **N**, how many times would we have to find the min to sort the list?
- **N**

- What is the run time of this sorting algorithm?
- $\mathbf{N}^2$

Quicksort Run Time

- For a list of size **N**, how many steps does it take to move everything less than the "pivot" to the left and everything greater than the "pivot" to the right?

- **N**

Quicksort Run Time

- How many times will the algorithm divide the list in half?
- **`lg(N)`**

- What is the run time of Quicksort?
- **`N * lg(N)`**

Different Run Times

- As our list gets bigger and bigger, which of the **search** algorithms is faster?
  - Linear or binary search?

- How much faster is binary search?
  - A lot!
  - But exactly how much is "a lot"?

Asymptotic Analysis

What is “Big O” Notation?

- Big O notation is a concept in Computer Science
  - Used to describe the complexity (or performance) of an algorithm
- Big O describes the **worst-case** scenario
  - Big Omega (Ω) describes the best-case
  - Big Theta (Θ) is used when the best and worst case scenarios are the same

A Simple Example

- Say we write an algorithm that takes in an list of numbers and returns the maximum
  - What is the absolute fastest it can run?
    - Linear time – **Ω(N)**
  - What is the absolute slowest it can run?
    - Linear time – **O(N)**
  - Are these two values the same?
    - YES – so we can also say it's **Θ(N)**

Simplification

- We are only interested in the growth rate as an "order of magnitude"
  - As the problem grows really, really, really large

- We are not concerned with the fine details
  - Constant multipliers are dropped
    - So `O(3 * N`^{2}`)` becomes simply `O(N`^{2}`)`
  - Lower order terms are dropped
    - So `O(N`^{3}` + 4N`^{2}`)` becomes simply `O(N`^{3}`)`

Asymptotic Analysis

- For a list of size **`N`**, linear search does **`N`** operations. So we say it is **`O(N)`** (pronounced "big Oh of n")

- For a list of size **`N`**, binary search does **`lg(N)`** operations, so we say it is **`O(lg(N))`**

- The function inside the **`O()`** parentheses indicates how fast the algorithm scales

Worst Case vs Best Case

- Why differentiate between the two?

- Think back to selection sort
  - What is the best case for run time?
  - What is the worst case for run time?
- They're the same!
  - Always have to find each minimum by looking through the entire list every time – $\Theta(N^2)$

Bubble Sort Run Times

- What about bubble sort?
  - What is the best case for run time?
  - What is the worst case for run time?

- Very different!
  - Best case, everything is already sorted – **Ω(N)**
  - Worst case, it's completely backwards – **O(N^2)**

Quicksort Run Times

- What about quicksort?
  - Depends on what the “hinge” or “pivot” is
- This determines how many times we split
  - But each split, we’ll need to compare each item to the hinge in their respective part: **O(N)**

- Best case, pivot is exact center – **Ω(N*lgN)**
- Worst case, it’s an “edge” item – **O(** N^2 **)**

Worst-case vs Best-case

- This is why, even though all three sorting algorithms have same worst case run times...
  – Quicksort often runs very, very quickly
  – Bubble Sort often runs much faster than Selection

- How does this apply to linear search and binary search? What are the best and worst run times for these?

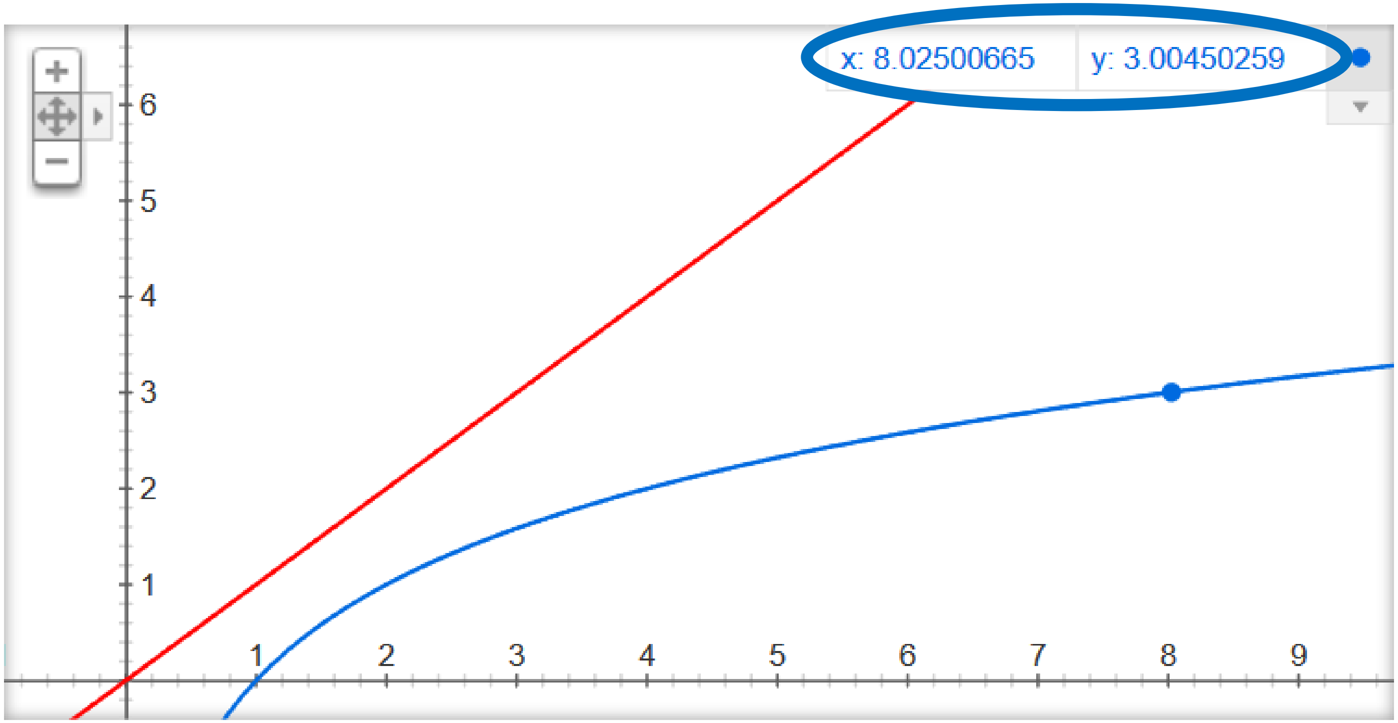
Search Run Times

- Linear search:
  - Best case: **Ω(1)**
  - Worst case: **O(N)**

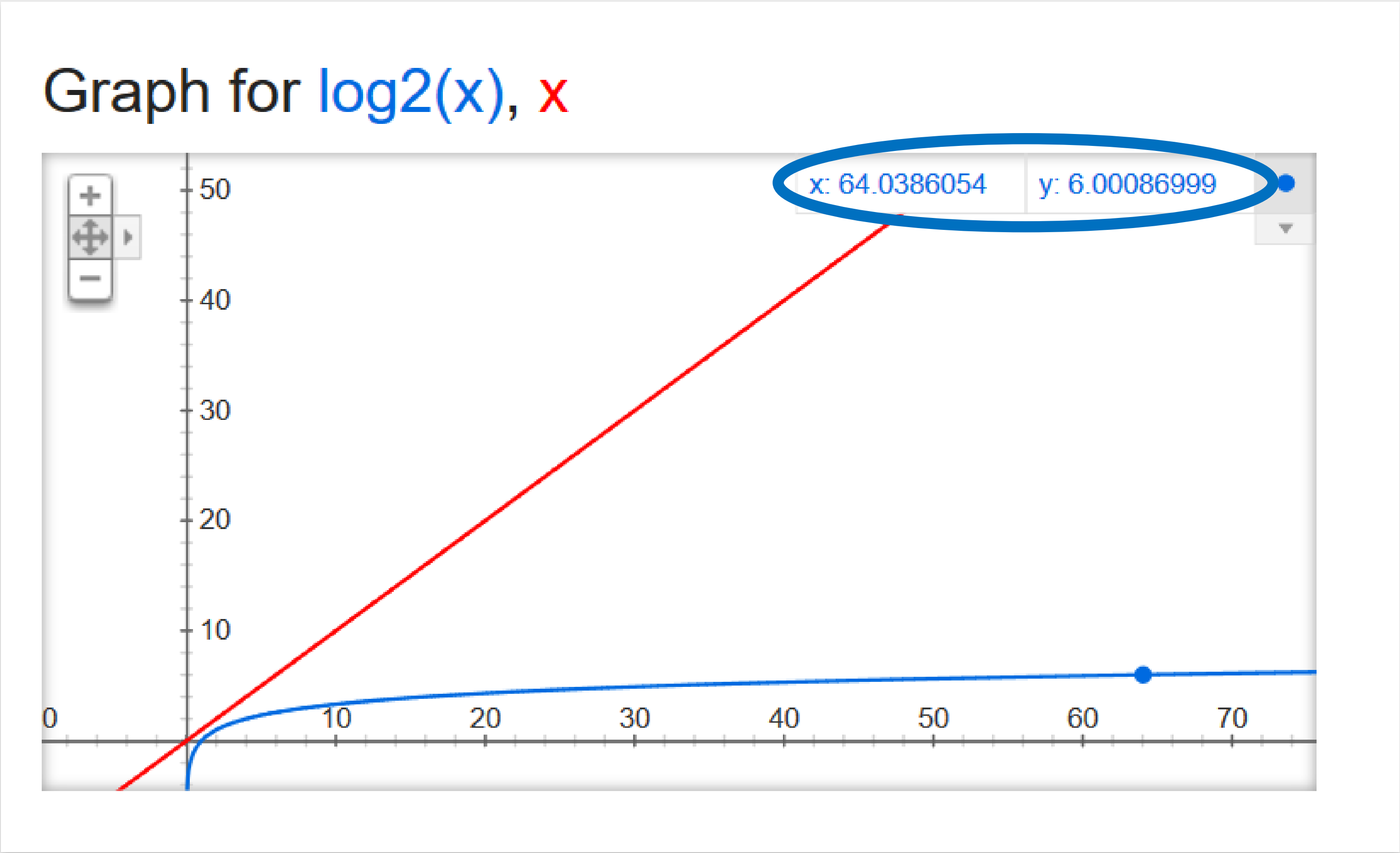
- Binary search:
  - Best case: **Ω(1)**
  - Worst case: **O(lg(N))**

Why Care?

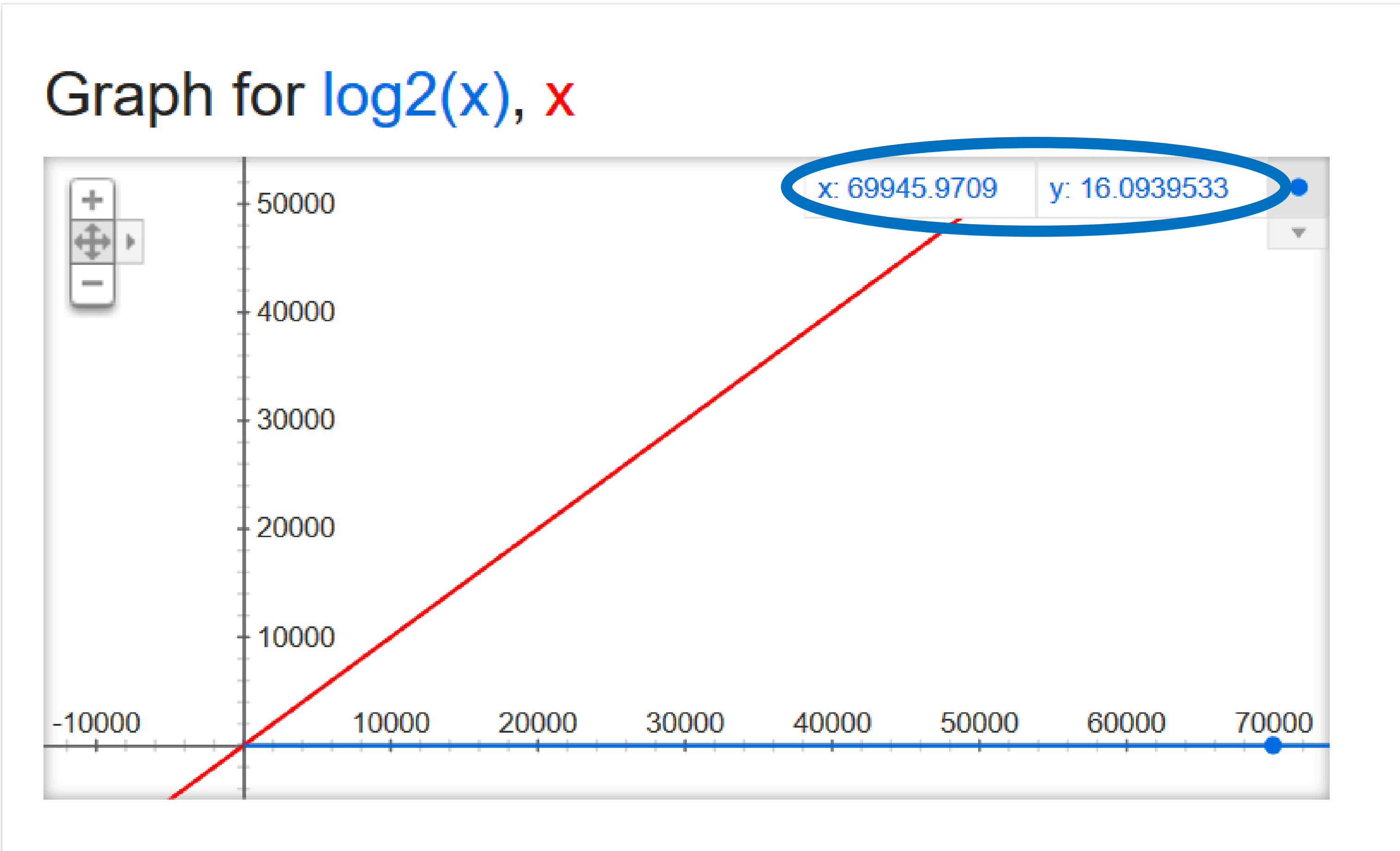
Graph for log2(x), x



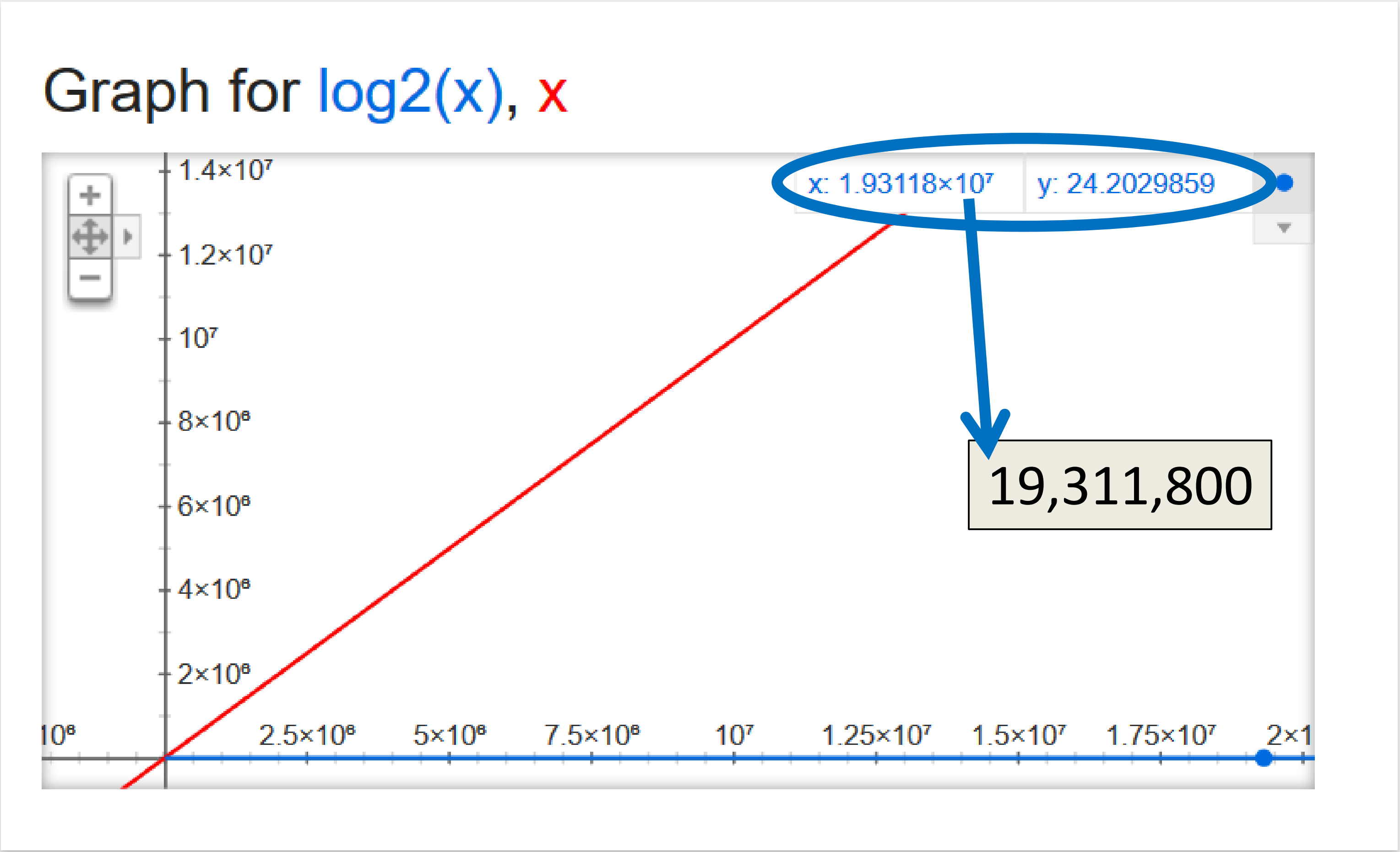
Why Care?


Graph for log2(x), x
x: 64.0386054
y: 6.00086999
50
40
30
20
10
0
10
20
30
40
50
60
70

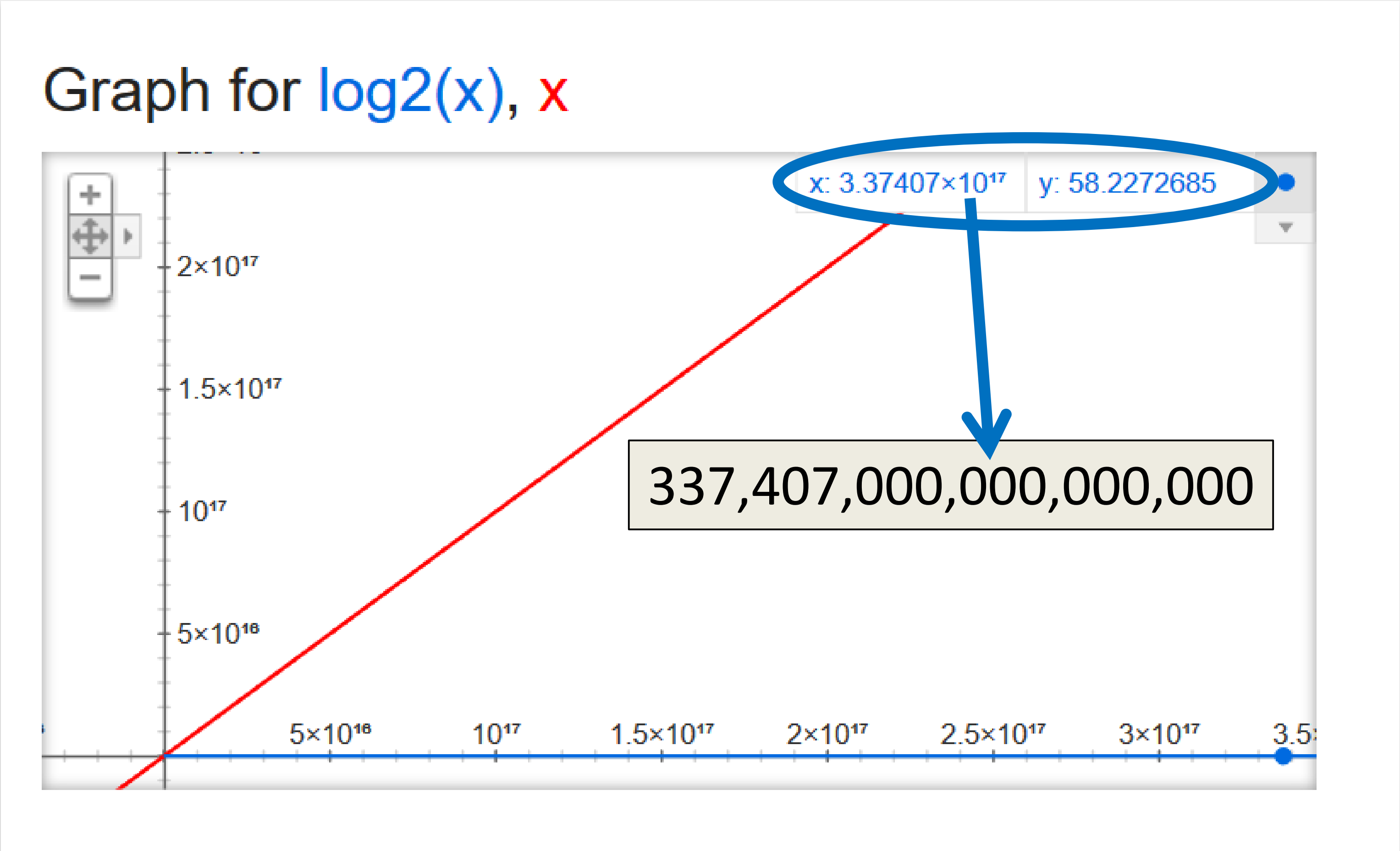
Why Care?


Graph for log2(x), x
x: 69945.9709
y: 16.0939533
50000
40000
30000
20000
10000
-10000
10000
20000
30000
40000
50000
60000
70000

Why Care?


Graph for log2(x), x
x: 1.93118×10⁷
y: 24.2029859
19,311,800
1.4×10⁷
1.2×10⁷
10⁷
8×10⁶
6×10⁶
4×10⁶
2×10⁶
2.5×10⁶
5×10⁶
7.5×10⁶
10⁷
1.25×10⁷
1.5×10⁷
1.75×10⁷

Why Care?


Graph for log2(x), x
x: 3.37407×10¹⁷
y: 58.2272685
2×10¹⁷
1.5×10¹⁷
10¹⁷
5×10¹⁶
5×10¹⁶
10¹⁷
1.5×10¹⁷
2×10¹⁷
2.5×10¹⁷
3×10¹⁷
337,407,000,000,000,000

Why Care?

- For large problems, there's a *huge* difference!
- If we can do 1,000,000 operations per second, and the list is 337.4 quadrillion items
  - Binary search takes 0.000058 seconds
  - Linear search takes 337,407,000,000 seconds
    5,623,450,000 minutes
    93,724,166 hours
    3,905,173 days
    10,699 years

Daily CS History

- Hedy Lamarr
  - Film star in 1930s - 1950s
  - Patented a frequency-hopping system that would make radio-guided torpedoes hard to detect or jam during World War II
  - Technologies like Bluetooth and Wi-Fi use similar methods

Announcements

- Project 3
  - Design is due ~~Tuesday,~~ Wednesday December 4th
  - Project is due Tuesday, December 11th
- Course evaluations also out, please complete

- Final exam is when?
  - Friday, December 14th from 6 to 8 PM
  - Locations will be posted on the course website
  - Common final

Image Sources

- Alphabetizing a Bookshelf video screenshot:
  - https://www.youtube.com/watch?v=WaNLJf8xzC4
- Graphs of x and $\log_2(x)$ courtesy of Google equation grapher
- Hedy Lamarr:
  - https://commons.wikimedia.org/wiki/File:Hedy_lamarr_-_1940.jpg